S = total dissolved sulfur dioxide (sulfurous acid, sulfite, and bisulfite), g moles/kg of water

Greek Letters

γ = activity coefficients

Component Subscripts

1 = sulfurous acid
2 = hydrogen ion
3 = bisulfite
4 = sulfite
5 = carbonic acid
6 = bicarbonate
7 = carbonate
8 = hydroxide
9 = sulfate
10 = sodium ion
11 = water

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Decoupling Control

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In a recent communication, Waller (1974) discusses the decoupling control of the transfer matrix or P-structure process model. As noted earlier (Changlai and Ward, 1972), the decoupling and primary feedback control roles for this particular process model cannot be separated. For this reason, there are an infinite number of decoupling choices, each resulting in different dynamic responses for the decoupled subsystems. A specified subsystem response can be approached only through a trial-and-error application of the diagonalization condition (Kavanagh, 1958; Freeman, 1957; Bollinger and Lamb, 1965). It may be difficult to even find a realizable design. There have been a number of attempts to provide straightforward control design for this terminal model by approximate methods.

PSEUDO-DECOUPLING

In an attempt to separate the decoupling and feedback roles, Waller proposes a simple feedforward decoupling logic, illustrated in the matrix structures of Figure 1. The diagonalization condition is applied to the matrix product $T = GD$. This decoupling implies that the pseudo-inputs m will not interactively affect the outputs y. However, the process intercoupling between the actual manipulative inputs u and the outputs y has not been eliminated. An additional difficulty is the effective design of the diagonal feedback control matrix P operating on pseudo-inputs rather than the process inputs since each p control element affects all of the u process inputs.

This pseudo-decoupling control logic is illustrated for a two-variable example in the signal flow diagram of Figure 2a, which corresponds to the block diagram (Figure 2) of the cited reference. The control structure in Figure 2a is that found in the feedback control of the transfer matrix model, as illustrated in Figure 2b. The variable v is used here to represent either a setpoint or disturbance input. Even though the setpoint enters the loop after y, this simplification is adequate for illustration since the control elements are actuated by the error.

DESIGN APPROACHES

Waller discusses two design classes for his proposed control structure. In the first, an arbitrary diagonal form

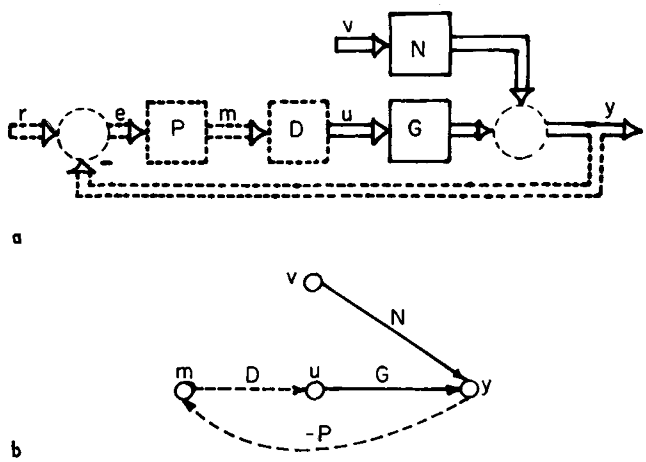

Fig. 1. Pseudo-decoupling control structure: (a) matrix block diagram, (b) matrix signal flow diagram.

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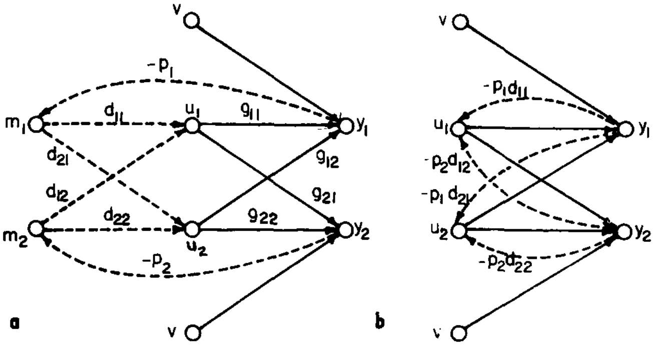

Fig. 2. Pseudo-decoupling for two-variable problem: (a) as proposed in cited reference, (b) equivalent transfer matrix feedback.

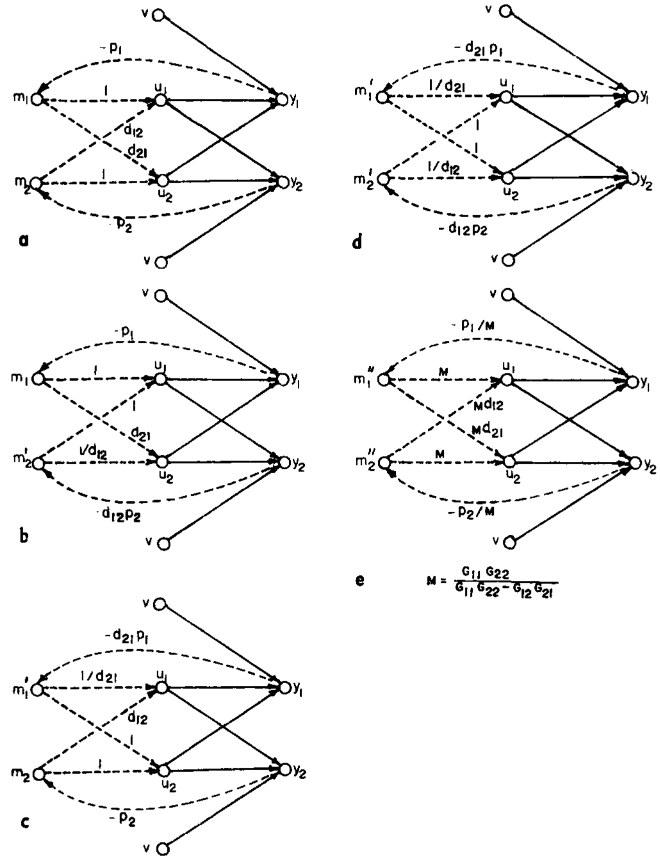

Fig. 3. Equivalent pseudo-decoupling control structures: (a) Equation (4) logic, (b) Equation (5) logic, (c) Equation (6) logic, (d) Equation (7) logic, (e) Equation (9) logic.

is chosen for the $T = GD$ matrix, followed by a realization of the resulting D matrix and tuning of the P loop controllers. The author mentions two arbitrary T-diagonalization choices based on the work of Zalkind (1967) and Luyben (1970).

In the second design class of Waller, n elements from different columns of the $n \times n$ D matrix are arbitrarily chosen first as simple forms. The remaining $(n^2 - n)$ elements of the D matrix are then fixed by the T diagonalization condition. It is again assumed that a suitable arbitrary form can be selected for the P loop controllers and the parameters tuned to give the desired response. The author lists the four combinations (Equations (4) to (7), cited reference) for a second-order system when the two arbitrary elements are both selected as unity. These four structures, as well as that corresponding to Equation (9) in the cited reference, are illustrated in Figure 3. Note that these equivalent structures can be related by graphic algebra. For example, Figure 3a can be transformed to Figure 3b by multiplying all branches entering m_2 by d_{12} and dividing all branches leaving m_2 by d_{12}. While the node m_2 is changed to a different variable m_2', the information flow is the same. As the author partially notes, there is an explicit relationship between the P loop controllers of all five equivalent cases. The relative ease of implementation of these P loop controllers is not evident from the illustrations and will depend on the form of P chosen. For example, if the loop controllers P/M in Figure 3e were labeled P^*, the loop combinations in the other choices would appear cumbersome in terms of P^*.

There are an infinite number of possible T diagonalization choices for the first class and an infinite number of D choices for the second class. Since there is no way to separate the decoupling and feedback roles for this process structure, there is no straightforward way to select effective choices in either case.

DISCUSSION

The decoupling control proposed by Waller is simply a feedback control matrix for the transfer matrix process model. It suffers from several weaknesses:

1. The method does not decouple the process outputs y from the process inputs u. Decoupling is achieved only in terms of the pseudo-inputs m.
2. Even if decoupling in terms of pseudo-inputs could be shown to be desirable, there are an infinite number of decoupling choices for the proposed control structure. The effectiveness of a particular choice would depend on the form of the loop controllers and there is no straightforward selection procedure.
3. The feedforward diagonalization approach neglects the feedback intercouplings between state variables. This intercoupling, which is implicit in the transfer matrix model, is evident in the state vector differential equation model.

The logic proposed by Waller is designed to reduce interaction effects due solely to setpoint changes. It can be viewed as a semi-empirical approach to the design of a feedback control matrix that might happen to give a suitable response and less interaction in simple cases by skillful selection of the P loop controllers. However, it does not provide a sound basis for the design of multivariable uncoupling control structures.

Two multivariable approaches for the feedback control of the transfer matrix model that might be considered are modal control and V-structure decoupling control. In modal control (Rosenbrock, 1962), a set of pseudo-variables are logically defined so as to facilitate the feedback control design. Limitations and extensions of the approach are discussed elsewhere (Rosenbrock, 1969; Changlai et al., 1973). V-structure decoupling control (Mesarovic, 1960) rearranges the process model form so that feedback decoupling and primary feedback control can be independently designed. The primary feedback control, whose form and parameters are not specified, is designed by conventional feedback methods and offers flexibility in response specification. An extension of the approach and a discussion of some limitations have been made by Greenfield and Ward (1968). If realizable approximations of the decoupling controllers are difficult to achieve due to time delays or the staged nature of the process, then the designer can either (1) resort to feedforward disturbance compensation along with decoupling, or (2) use the state vector differential equation model to design the control.

If the differential equation model is reasonably well identified, as is true in many staged operations, there is little incentive to use a transfer matrix model for control system design. A variety of methods are available for the state vector model. These include uncoupling approaches such as structural-analysis (Greenfield and Ward, 1967) and state variable feedback (Falb and Wolovich, 1967), as well as optimal control synthesis (Schuldt and Smith, 1971) and modal approaches (Changlai et al., 1973).

NOTATION

D = decoupling matrix
d = decoupling matrix element
G = process transfer matrix
g = process transfer matrix element
M = matrix defined in Figure 7
m = pseudo-input vector
P = diagonal feedback control matrix
p = feedback control matrix element
r = setpoint vector (see v definition below)
T = matrix product GD
u = process input vector
v = process disturbance vector (This symbol is used to represent either a disturbance or setpoint input in signal flow graphs.)
y = process output vector

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Reply

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The above note by Yang and Ward is essentially an extract from a previous note by Changlai and Ward (see Literature Cited above). The control strategy labeled *decoupling* by Changlai and Ward is labeled *pseudo-decoupling* by Yang and Ward, who expect this change of nomenclature to lead to difficulties in the implementation of the control. I don't.

The main point in the note by Yang and Ward is that in distillation *V*-structure design is to be preferred to *P*-structure design. Rijnsdorp (1965) has come to the opposite conclusion. I join Rijnsdorp and to his motives I would like to add the following:

One very serious drawback of connecting the decouplers according to the *V*-scheme of Changlai and Ward is that they easily may turn out to be unrealizable, in distillation probably more often than not. In the experimental example discussed in my note, for instance, the *V*-decouplers are unrealizable since they have to contain negative dead time. This is a consequence of the fact that they have been so unsuitably connected that dead time in the process has to be compensated by negative dead time in the decouplers. This drawback is far too great to be compensated by the advantage that only two decouplers are needed (for systems with two inputs and two outputs). Another drawback of the *V*-design, in line with the argumentation of Yang and Ward, is that the design results in only one out of the infinite number of possible decoupling schemes and that nothing guarantees that this special choice is more effective than other choices.

The four simple schemes discussed in my note, Equations (4) to (7), also need only two decouplers (for two input-two output systems). But besides providing straightforward and simple design, they have the advantage of being related in such a way that at least one of the schemes is realizable with good approximation in practically all cases.

Yang and Ward consider the lack of uniqueness in *P*-structure design a weakness. The example discussed above illustrates that freedom in design is a strength and not a weakness.

What is left in the above note by Yang and Ward—besides some self-evident explanations of simple facts in my note—is the statement that there are several other design methods than decoupling for multivariable control systems. With this I completely agree.

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